Exploring Geometric Conjectures with the help of a Learning Environment - A Case Study with Pre-Service Teachers

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Abstract

Several researches show the potential of using new information and communication technologies (ICT) in the teaching and learning of mathematics. One of the branches in the teaching of mathematics that, over time, has deserved a special mention, is geometry, namely with the use of dynamic geometry systems (DGS). A learning environment for teaching and learning geometry, associated with tasks of an exploratory and investigative nature, tends to favor the discovery of properties and geometric relationships, which benefits the acquisition of knowledge and the production of evidence. In this article the learning environment Web Geometry Laboratory (WGL) was used in a case study for the exploration of visual proofs. The study was done with pre-service teachers. Conclusions are drawn and future work is foreseen.

1 Introduction

Mathematical proof remains a means to everyone to prove to someone the truthfulness of an out tasks [5, 9, 17]. On the one hand we have a formal axiomatic theory, with a clear set of axioms (set of truths about a given reality, organized into concepts based on primitive terms) and lemmas, where geometric conjectures can be formally proved, on the other hand we have geometric models, geometric constructions with its appealing, intuitive, visual rendering. Dynamic geometry offers the ability to experiment, to use strategies, guess, argue and deduce mathematical properties. The concrete manipulation of objects enables abstract manipulation, thus making more accurate deductions that lead to the development of mathematical reasoning [17]. The use of geometric construction to infer a given geometric property can be misleading, the construction is only a particular case of the geometric property that is being considered. The use of an e-learning environment incorporating a dynamic geometry system (DGS) for the learning of geometry is, in our opinion, a very important help, it will facilitate the learning process. The DGS allow to, dynamically, explore many instances (constructions) for a given conjecture, it is not a formal proof yet, but it encourages the elaboration of conjectures, helping the progress in mathematical communication, and developing mechanism of deductive reasoning.

With the advances in the distance education support tools and the Internet, it has become possible to disseminate knowledge extremely quickly and to meet the request of courses with flexible time and location. In this context, virtual environments become classrooms, where students and teachers communicate and interact through resources such as chats, emails and collaborative tools.

1.1 Collaborative Learning

Collaborative learning is defined as an approach to teaching and learning, involving groups of students working together, to develop a set of interpersonal skills, such as effective communication, negotiation, conflict resolution, decision making, leadership, personal responsibility and teamwork, exchanging knowledge and experiences, to solve a given problem, complete a task, or create a product [1, 16]. It is a possible strategy for different levels of education [29]. With the development of technologies, learning environments are enriched tools that allow sharing experiences and communications among its users. Collaborative learning computational systems become central to online education [24]. Collaborative work favours training in the capacity for synthesis, critical thinking, and the ability to summarize ideas or conjectures. The new technologies, in general, assume a preponderant role in other areas, with emphasis on the development of the problem-solving capacity due to the versatility that they show in the approach of different situations

1.2 Dynamic Geometry Tool

The role of information and communication technology (ICT) in the classroom had increased in the recent years [28]. The use of ICT tools represents a technological support for the visualisation of abstract concepts, allowing the production of mental models of the concept and assuming a more active role of the student in her/his learning process. With computer programs, students interact with educational material to develop the skills needed to solve problems using mathematics. In the area of geometry, the DGS are already well-known tools, steadily, but surely, substituting the ruler and compass tools. Many different DGS are currently available, e.g. *Cabri* [18], *C.a.R.* [10], *Cinderella* [23], *GeoGebra* [11] and *The Geometer's Sketchpad* [12][1].

One of the advantages of dynamic geometry programs is the accomplishment of tasks, not only exploring geometric situations, but also investigating situations that the tool itself fosters when moving objects, providing valuable support for students and teachers.

A case study for the exploration of visual manipulations and the formal validation counterpart is presented in this article. The *Web Geometry Laboratory (WGL)* [21], an e-learning collaborative and adaptive Web environment for geometry, incorporates a DGS, with a support of a database where each user can save geometric constructions produced using the DGS. The *WGL*, with the incorporated DGS, *GeoGebra 5* [11], is used to explore the many possible configurations for a given geometric construction and its properties, but also to introduce the formal validation of those properties using the new `Prove` and `ProveDetails` commands of *GeoGebra.*

1.3 Automated Theorem Proving Tool

Geometry Automatic Theorem Provers (GATP) are computer programs used to formally prove theorems in geometry, e.g. *GCLC* [13] and *JGEX* [30][2]. A GATP processes a series of conjectures, hypotheses and axioms, written in a formal language that allows to express in a precise and unambiguous way the problem to be solved, to generate a proof that describes how and why the conjectures follow from the axioms and hypotheses in a way that can be understood by humans. The GATP can be seen as computer programs that shows whether a sentence, a conjecture, is a logical consequence of a set of axioms and hypotheses. The proof describes a sequence of steps that validates the conjecture. Some pioneering work in the development of automatic prover of geometry theorems were made by Gelernter in the 60s of the 20th century [8], developing in a very active area of research [2, 3, 20]. Demonstrations are indispensable for the expansion of mathematical knowledge; the simple

[1] https://cabri.com/; http://car.rene-grothmann.de/; https://www.cinderella.de/; https://www.geogebra.org/; http://www.dynamicgeometry.com/

[2] http://poincare.matf.bg.ac.rs/~janicic/gclc/; https://sourceforge.net/projects/jgex/

act of planning a proof contributes to the development of mathematics. Proofs produce new mathematical views, new contextualized connections, and new methods for solving problems, giving them a value far beyond proving the veracity of propositions. In secondary school textbooks, the proofs in mathematics are introduced in contents related to geometry.

From the point of view of the implemented case study the focus was in the use of a DGS with an integrated GATP, allowing, in this way, the formal validation of a given conjecture, i.e. to explore a given geometric conjecture from the point of view of the DGS, by exploring, dynamically, the geometric conjectures, but, at the same time, having the possibility, through the integrated GATP, to have a formal validation of the given conjecture. More specifically, using *WGL*[3] and its integrated DGS, *GeoGebra 5*, the use of the command line tool was used to make a formal validation of a given conjecture. *GeoGebra* has two validation tools, the $a \stackrel{?}{=} b$ tool performs a numerical check, but in certain extreme circumstances, numerical checks can lead to false results [14, 15]. The other tool is the command `Prove`, this tool uses the incorporated GATP to formally prove some geometric conjectures, e.g. parallelism, equality, perpendicularity, and collinearity.

The current study with pre-service teachers aimed to dealing with human reasoning with technology, that is, how students accomplish their mathematical work in a learning environment. How they are dealing, in groups, with automatic theorem proving in dynamic geometry systems?

2 The Experiment Framework

In the beginning of the 2nd semester of the academic years 2018/2019 and 2019/2020, a questionnaire was given. The first section of the questionnaire concerned the characterization of the respondents with respect to age and gender. The second section of the questionnaire concerned digital technologies and consisted of 32 statements to which students had to give their degree of agreement in a five-point Likert scale. The third section of the questionnaire about collaborative environments (based on the Constructivist On-Line Learning Environment Survey) [27] was applied to get an initial students profile in relation to beliefs about reflective thinking and learning from other students. The last part of the questionnaire was given to help assess key questions about the quality of an online learning environment from a social constructivist perspective. The last part of questionnaire consists of 18 questions arranged in 5 scales:

- Relevance - how relevant is online learning to students' practices?
- Reflection - does online learning encourage critical reflection by students?
- Interactivity - to what extent do students participate online in dialogues with an educational interest?
- Teacher Support - how can tutors' activities enable students to participate in online learning?
- Peer Support - do colleagues provide sensitive and encouraging support?

2.1 Questionnaire Analysis

The student's answers to the questionnaire were analysed in order to get a perspective about their skilfulness in the use of digital technologies and collaborative environments.

[3] https://hilbert.mat.uc.pt/WebGeometryLab

Table 1: Measure of Student's ICT Skills.

Question number	Question
Part II–11	I can implement effective classroom management in teaching and learning processes when using technologies
Part II–13	I can use technologies to implement various educational activities such as homework, projects, etc.
Part II–29	I can solve problems that can be found in online educational environments (e.g. Moodle)
Part III–2	Relevance (I prefer that, …)
Part III–5	Tutor support (I like when teacher …)
Part III–6	Peer support (I prefer when colleagues …)

The students' answers: *about implementing technologies in the classroom*, 61.5% totally agree or agree and 38.5% neither agree nor disagree; *about the use of technologies to implement various educational activities such as homework, projects, etc.*, 69.2% totally disagree or disagree and 30.8% neither agree nor disagree*; about online educational environments*, 30.8% totally agree or agree, 38.4% neither agree nor disagree 30.8% and totally disagree or disagree (see Figures 1–3).

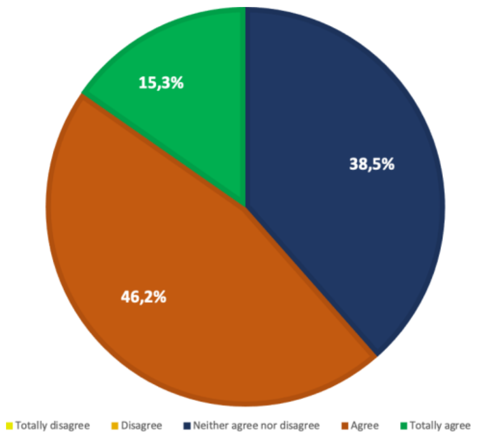

Figure 1: Implementing technologies in the classroom

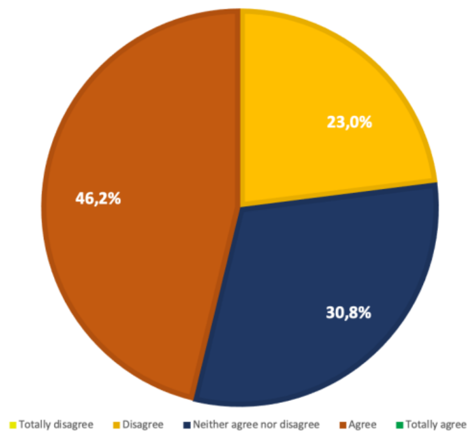

Figure 2: Use of technologies to implement various educational activities such as homework, projects, etc.

The last part, on collaborative environments, 74.4% of students agree, always, *with the relevance of the use of collaborative environment, that helps them to improve knowledge* and 100% agree always *when teacher stimulate their reasoning*. They prefer always *the teachers' help* 66.6% against 48.1%.

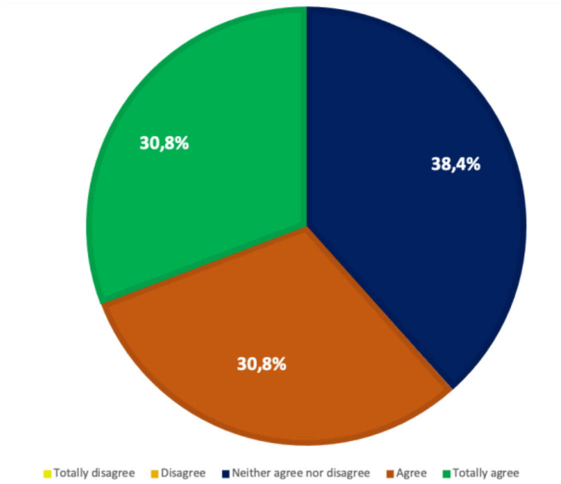

Figure 3: Online educational environments

colleagues' help, they *felt supported and encouraged by the online support provided by their teachers*, responding 11.5% who almost never prefer *the help of colleagues*.

2.2 Activity Description

Three activities were proposed to the students of University of Aveiro master's programme for Pre-service Teacher, *MSc for Training of Teachers for the 1st to 6th grades with emphasis on Mathematics and Natural Sciences*, during the 2nd semester of academic years 2018/2019 and 2019/2020, in line with the basic education curriculum, within the topic of geometry and measurement.

The curriculum states that activities should enable the development of visualisation skills and the construction of mathematical explanations and justifications and logical reasoning, including the use of examples and counterexamples.

These students have already had contact with the *GeoGebra* in other curricular units, namely in *Further Mathematics and Didactics of Mathematics in Basic Education*. Previously, before we started with this activity, there was a session between students on the use of *WGL*, collaboratively. About *GeoGebra*, it was just a warm-up about tool bar and *GeoGebra* commands, as they already know this DGS during classes in other curricular units.

It remains to be noted that the academic year 2018/2019 was in a face-to-face context and the academic year 2019/2020 was in the remote context, because of the pandemic Covid-19. From March 16 onward classes were given online.

Activity With this activity, a given construction (see Figure 4) was provided. The construction protocol is:
Given two points, *A* and *B*, and a line through them. Give a point *D* and a line through *D* and parallel to line *AB*. Point *E* on line *DF* (point *F* appears when create the parallel line). Given a line through *B* and *E*.

The questions placed to the groups were:

1) Verify that the AC and DF lines are parallel;
Using letters from the figure, show:
1.1 Two parallel lines;
1.2 Two lines directly parallel;
1.3 A pair of corresponding angles;
1.4 A pair of alternating internal angles;
1.5 A pair of alternating external angles;
1.6 Two vertically opposite angles;
1.7 Two supplementary adjacent angles;
2) Do you keep the same answers if the AC and DF lines are not parallel?

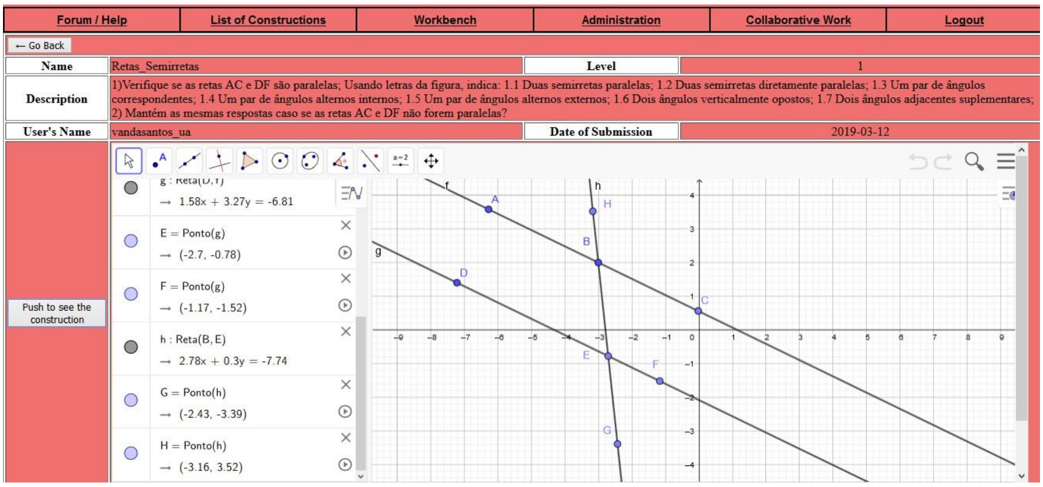

Figure 4: Previous Construction - List of Constructions

The constructions produced by groups 1, 2 and 3 can be seen in figure 5 to 11, respectively.

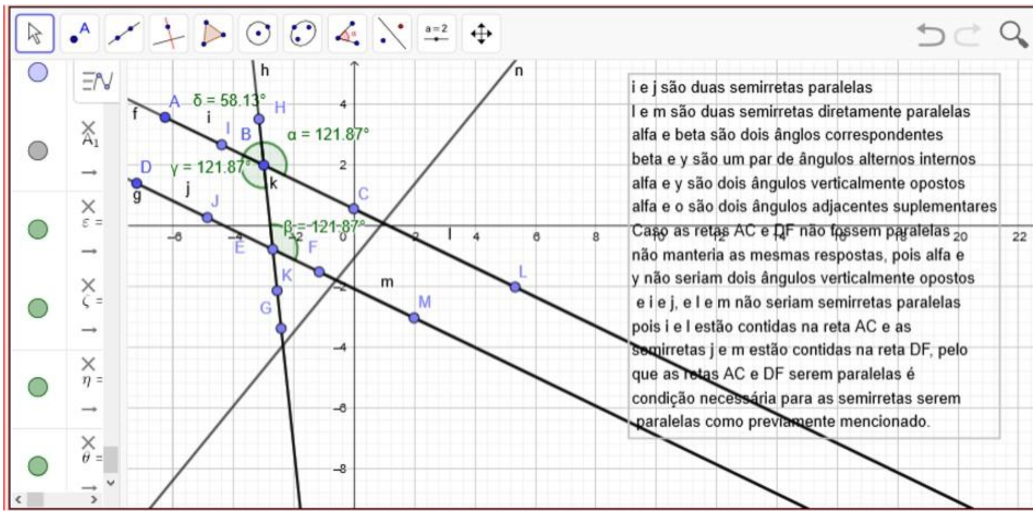

Figure 5: Group 1 – Academic Year 2018-2019, question 1

Figure 6: Group 1 – Academic Year 2018-2019, question 2

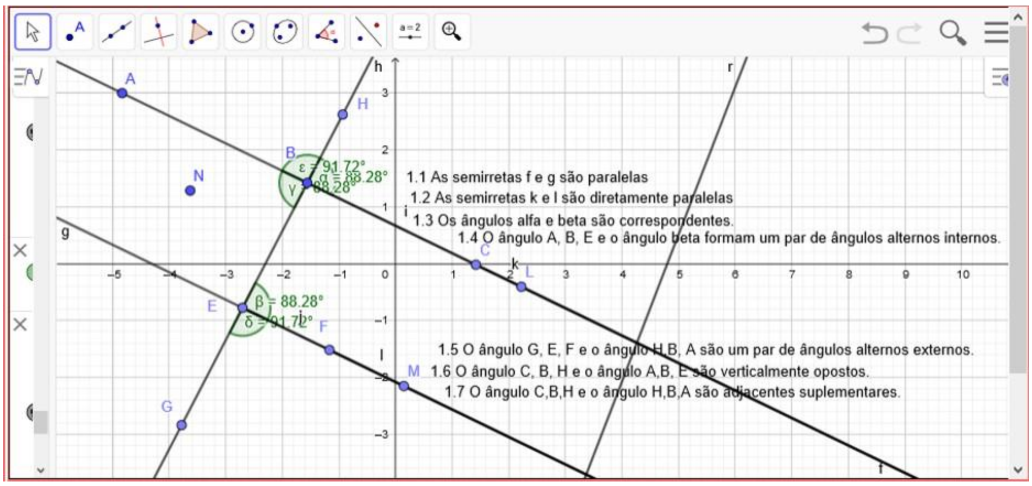

Figure 7: Group 2 – Academic Year 2018-2019, question 1

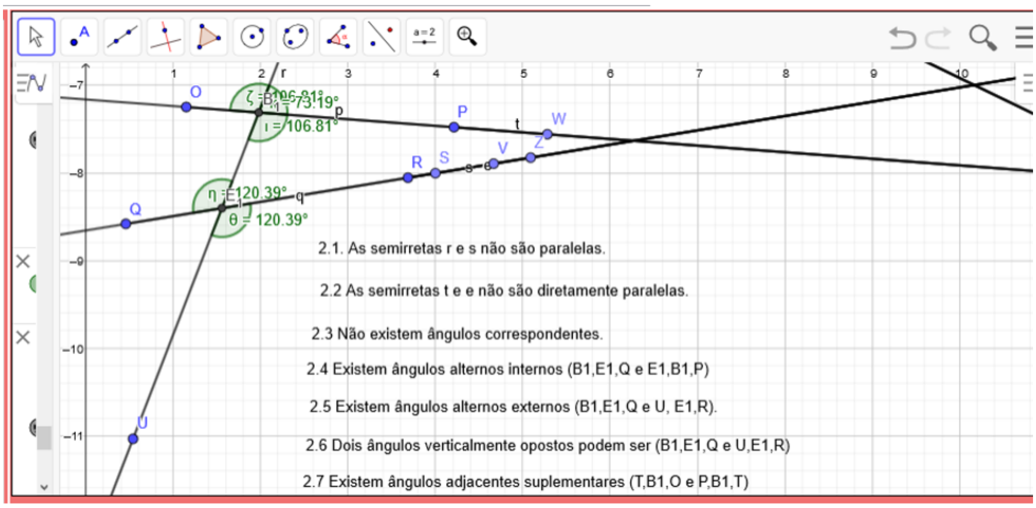

Figure 8: Group 2 – Academic Year 2018-2019, question 2

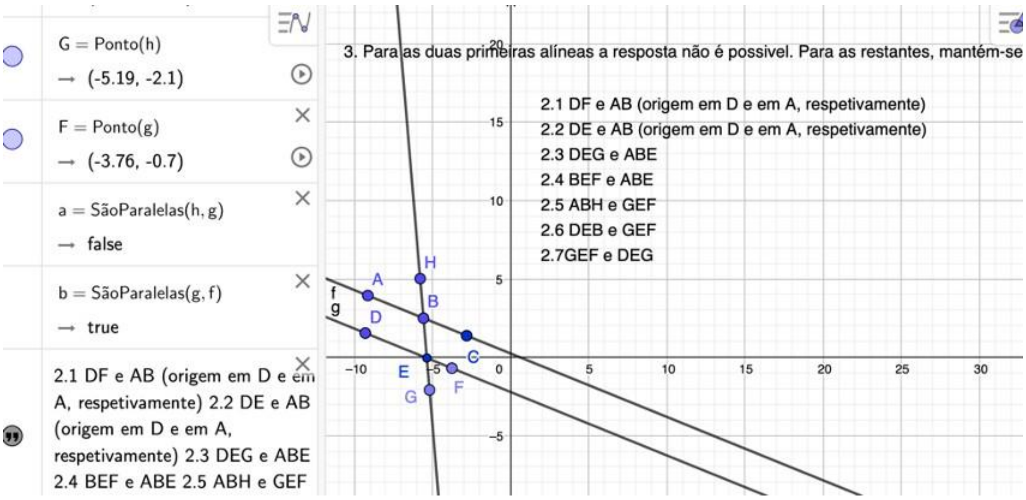

Figure 9: Group 1 – Academic Year 2019-2020, questions 1 and 2

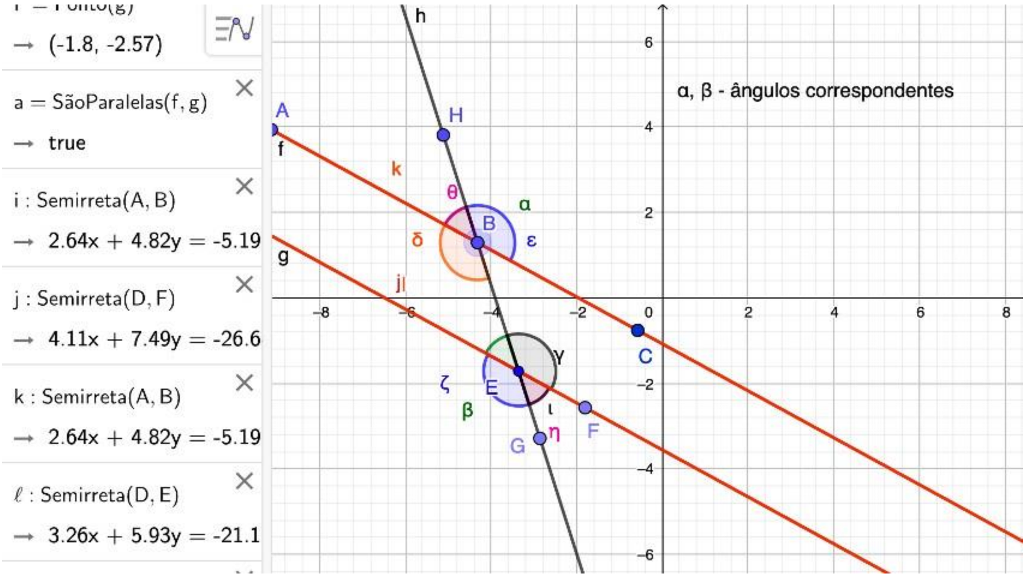

Figure 10: Group 2 – Academic Year 2019-2020, questions 1 and 2

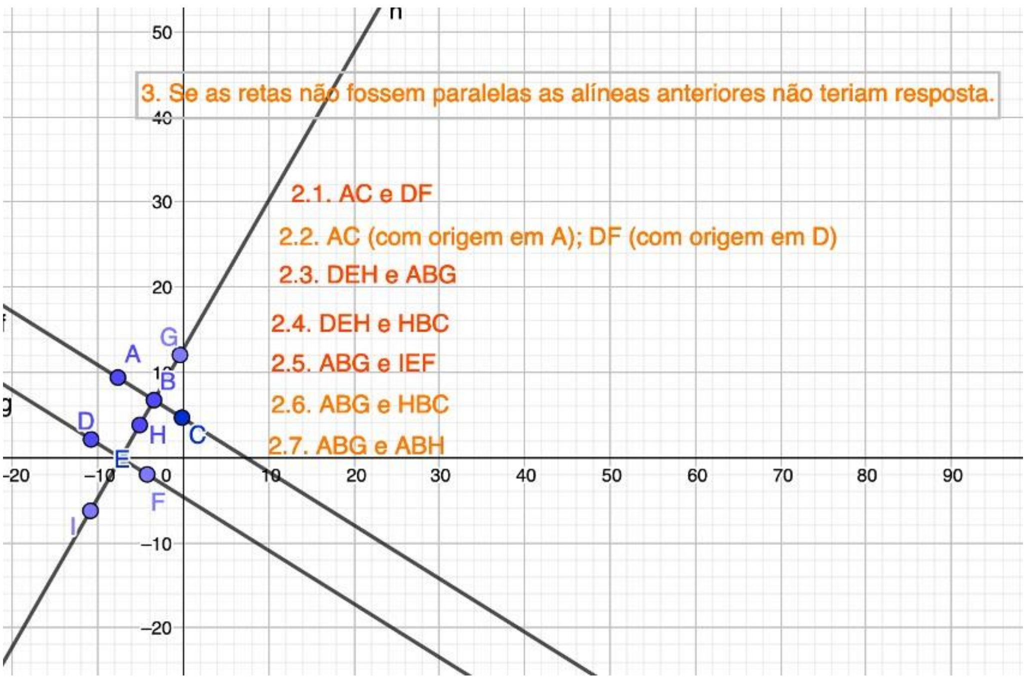

Figure 11: Group 3 – Academic Year 2019-2020, questions 1 and 2

For the first question the answers of the groups differ. Although both groups used the command `Prove(AreParallel(<line>,<line>))`, they did not use the command, `Prove(AreEqual(<line>,<line>))`. They tried to justify their conclusions by visual means, not using the proving tool.

Interactions at WGL We show below excerpts of chat conversations between students in the selected class (Group 1 - Academic Year 2018-2019), such as text transcripts of students who interacted with other group colleagues during a given task resolution. We can see the interaction between MCII_52 and MCII_62 and deduce that MCII_62 is trying to figure out the steps needed to solve the task.

1 – MCII_52: Why all the angles have the same magnitude?
2 – MCII_52: The gamma angle at 1.3 was right?
3 – MCII_62: It's true what it says at the input?
4 – MCII_62: Sends response to the group.
5 – MCII_52: Which lines *f* and *g* is for which? 1.2.?
6 – MCII_62: I unlocked it
7 – MCII_62: But this is 1.1. I can't, just give the lines *f* and *g*
8 – MCII_62: OK. I already wrote that from the angles.

The interactions between the Group 2 - Academic Year 2019-2020.

1– MCII_3: Hi!
2– MCII_3: I can start if you want.
3– MCII_4: Yes, please.
4– MCII_4: How do you see the command?
5– MCII_3: Already managed to select the input command.
6 – MCII_4: Did you get it?
7 – MCII_3: They are parallel

On the one hand the collaboration allowed the students to help each other in solving the proposed task. Collaboratively, the students shared the construction in the group, thus exchanging the geometric information among themselves. On the other hand, we can see what they said between them and we can observe the constructions made by them, for example at figure 11 it is possible to see that the group, in order to the activity, used the values of the angles to compare between them. This comparison is only numerical, and it do not constitute a formal proof, although this comparison was not requested in the activity.

3 Conclusions

This first case study suggests that the concept of formal proofs and its use in an actual classroom situation is still a challenge subject to pre-service teachers. We can see that in the chat transcript, some groups use the value of the angles that lines form with each other (line *h* with line *AC* or line *h* with line *DF*, at figure 4), to prove that the lines *AC* and *DF* are parallel, instead of using the *Prove* commands.

The use of a collaborative environment (chat and DGS) allowed students to work more efficiently, as it managed to reach the desired result, because they could see, and they were doing in the group area and communicate with each other on the activity.

The visualization is an intuitive help needed to introduce a demonstration, geometric reasoning depends solely on the available propositions, definitions, axioms and theorems, but on the other hand visualization can also be misleading (in a particular case, for example). The concept that the DGS dynamism can be used to test, but not to prove, a given geometric property and that, in order to prove it, a GATP must be used, it is still not familiar to them [22]. In the second activity they did not explore the command `Prove(AreEqual(<line>,<line>))`. A larger study must be conducted and strategies to address the problem must be though.

The use of technology to aid the teaching and learning process represents one of the areas in which applications of intelligent tutoring systems are being developed in recent times, such as *GRAMY* [19] and *QED-Tutrix* [6, 7]. The implementation, in the *WGL* [21, 25], of adaptive features that can help its users in the validation process is a project that will be pursued in the near future.

Knowledge of mathematics to teach is more than knowing mathematics for oneself, it is understanding concepts correctly, as well as performing procedures, but also being able to understand the conceptual foundations of these concepts and procedures. As stated in [4], if we want the aspects mentioned on reasoning and its processes to be worked on from the early years, those have to be worked on during initial training, because one of the factors identified in the literature as limiting this work is the teachers' poor knowledge about the same [26].

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